SAMPLE PAPER 2014: PAPER 1

QUESTION 2 (25 MARKS)

Question 2 (a) (i)

STEPS TO PROOF BY INDUCTION
1. Prove result is true for some starting value of $n \in \mathbb{N}$.
2. Assume result is true for $n = k$.
3. Prove result is true for $n = (k+1)$.

REQUIRED TO PROVE: $1+2+3+............+n=\dfrac{n(n+1)}{2}$

STEP 1: Prove the result is true for $n = 1$.

$$1=\frac{1(1+1)}{2}$$
$$1=\frac{1(2)}{2}$$
$1=1$ [Therefore, true for $n = 1$.]

STEP 2: Assume it is true for $n = k$.

$$\underline{1+2+3+............+k}=\frac{k(k+1)}{2}$$

STEP 3: Prove it is true for $n = k + 1$.

Prove $\underline{(1+2+3+............+k)}+(k+1)=\dfrac{(k+1)(k+2)}{2}$

Use the result in Step 2 to prove Step 3.

$$(1+2+3+............+k)+(k+1)$$
$$=\frac{k(k+1)}{2}+(k+1)$$
$$=(k+1)\left[\frac{k}{2}+1\right]$$
$$=(k+1)\left[\frac{k+2}{2}\right]$$
$$=\frac{(k+1)(k+2)}{2}$$

Therefore, assuming true for $n = k$ means it is true for $n = k + 1$. So true for $n = 1$ and true for $n = k$ means it is true for $n = k + 1$. This implies it is true for all $n \in \mathbb{N}$.

Question 2 (a) (ii)

$$S_n = \frac{n(n+1)}{2}$$

$$S_{100} - S_{50} = \frac{100(101)}{2} - \frac{50(51)}{2} = 3775$$ ← This the the sum of the numbers between 51 and 100.

Question 2 (b)

$$\log_c \sqrt{x} + \log_c (cx)$$
$$= \log_c x^{\frac{1}{2}} + \log_c (cx)$$
$$= \tfrac{1}{2} \log_c x + \log_c c + \log_c x$$
$$= \tfrac{3}{2} \log_c x + \log_c c$$
$$= \tfrac{3}{2} p + 1$$

Formulae and Tables Book
Indices and logs [page 21]

$$\log_a (xy) = \log_a x + \log_a y$$
$$\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$$
$$\log_a (x^q) = q \log_a x$$
$$\log_a 1 = 0$$
$$\log_a \left(\frac{1}{x}\right) = -\log_a x$$